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On the analysis of the pion–nucleon σ–term: The size of the remainder at the Cheng–Dashen point

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Abstract

We calculate the one–loop contributions of order M_π^4 to the difference Δ_R between the on–shell pion–nucleon scattering amplitude $\bar{D}^+(0, 2M_\pi^2)$ at the Cheng–Dashen point $\nu = 0$, $t = 2M_\pi^2$ and the scalar form factor $\sigma(2M_\pi^2)$ in the framework of heavy baryon chiral perturbation theory. We proof that to this order Δ_R contains *no* chiral logarithms and therefore it vanishes simply as M_π^4 in the chiral limit. Numerically, we find as an upper limit $\Delta_R \simeq 2\,\mathrm{MeV}$.

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1. Pion–nucleon scattering data allow to extract information on the size of the pion–nucleon σ–term, $\sigma(0)$, which measures the explicit chiral symmetry breaking in QCD due to the up– and down–quark masses. A venerable (current algebra) low–energy theorem due to Brown, Pardee and Peccei [1] relates $\sigma(0)$ to the isoscalar πN scattering amplitude (with the pseudovector Born term subtracted) via

$$F_\pi^2\,\bar{D}^+(0,2M_\pi^2) - \sigma(2M_\pi^2) = F_\pi^2\,\bar{D}^+(0,2M_\pi^2) - \Delta_\sigma - \sigma(0) = \Delta_R = M_\pi^4\,C_R \qquad (1)$$

with $F_\pi\,(M_\pi)$ the pion decay constant (mass) and $\Delta_\sigma = \sigma(2M_\pi^2) - \sigma(0)$. The crucial statement of the low–energy theorem is that the remainder Δ_R grows quadratically with the light quark mass. The on–shell πN scattering amplitude $\pi^a(q)+N(p) \to \pi^b(q')+N(p')$, with $s=(p+q)^2$ and $t=(p-p')^2$ the conventional Mandelstam variables, is defined in the standard fashion,

$$T^{ba}_{\pi N} = \bar{u}(p')\{\delta^{ba}[A^+(s,t) + \not{q}\,B^+(s,t)] + i\epsilon^{bac}\tau^c[A^-(s,t) + \not{q}\,B^-(s,t)]\}u(p)\;. \qquad (2)$$

We also introduce the variable $\nu = (s - m^2 + t/2 - M_\pi^2)/2m$, with m the nucleon mass. At the Cheng–Dashen point $\nu=0$, $s=m^2$, $t=2M_\pi^2$ [2],

$$\bar{D}^+(0,2M_\pi^2) = A^+(m^2,2M_\pi^2) - \frac{g_{\pi N}^2}{m}\;, \qquad (3)$$

with the last term due to the subtraction of the pseudovector tree amplitude. Furthermore, the nucleon scalar form factor $\sigma(t)$ is given by the matrix element

$$< N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p) >= \bar{u}(p')\,\sigma(t)\,u(p)\;,\; t=(p-p')^2\;. \qquad (4)$$

Although the Cheng–Dashen point is not in the physical region of the πN scattering process, it lies well within the Lehmann ellipse and thus $\bar{D}^+(0,2M_\pi^2)$ can be obtained by analytic continuation, i.e. using dispersion relations. The most recent analysis leads to $F_\pi^2\,\bar{D}^+(0,2M_\pi^2) = 60\,(62)\,\mathrm{MeV}$ for two sets of low energy πN data [3]. For a discussion of the uncertainties (typically ± 8 MeV) and previous determinations, we refer to [3]. The leading non–analytic contribution to the scalar form factor difference, Δ_σ, is $3g_{\pi N}^2 M_\pi^3/(64\pi m^2)$ and gives about 8 MeV [4] [5]. Evaluating the same one–loop diagram with an intermediate $\Delta(1232)$ isobar adds another 7 MeV [6]. A detailed dispersive analysis [3], with $\pi\pi$ and πN information consistent with chiral symmetry, yields $\Delta_\sigma = 15.2 \pm 0.4\,\mathrm{MeV}$.

The remainder Δ_R is not fixed by chiral symmetry. It has to be known, however, to extract information on the σ–term, i.e. $\sigma(0)$, and from it the strangeness content of the proton (the matrix element $< p|\,\bar{s}s\,|p >$). Let us briefly summarize what is known about the size of Δ_R. Brown et al.[1] estimated the remainder from tree level resonance excitation, with most of its contribution coming from the $\Delta(1232)$, of the order of 0.6 MeV. Furthermore, it was shown that spin–$\frac{1}{2}^\pm$ resonances do not contribute to the isoscalar πN amplitude at the Cheng–Dashen point while the higher spin–$\frac{3}{2}^-$ resonance contributions are suppressed by two orders of magnitude compared to the $\Delta(1232)$ [1]. Gasser et al. [5]

performed a complete one–loop calculation of the πN scattering amplitude in relativistic nucleon chiral perturbation theory to order q^3 and found $\Delta_R^{(\mathrm{GSS})} = 0.35\,\mathrm{MeV}$, or truncated at order M_π^4,

$$\Delta_R^{(\mathrm{GSS})} = \frac{g_A^4\, M_\pi^4}{32\pi^2 m F_\pi^2} = 0.46\,\mathrm{MeV} \ . \tag{5}$$

Therefore, the conjecture of Pagels and Pardee [4] that Δ_R contains potentially large logarithms of the form $M_\pi^4 \ln M_\pi$ could not be verified (to order q^3). However, there could still be large logarithms at one–loop in diagrams which have exactly one insertion from the dimension two effective pion–nucleon Lagrangian $\mathcal{L}_{\pi N}^{(2)}$. Such a large effect at subleading order has already been observed in the calculation of the magnetic polarizability of the proton, where at order q^4 the loop graphs generate a $\ln M_\pi$ term with a large coefficient which cancels most of the big contribution from the $\Delta(1232)$ encoded in the low–energy constant of the pertinent contact term from $\mathcal{L}_{\pi N}^{(4)}$. Furthermore, some of the coefficients of $\mathcal{L}_{\pi N}^{(2)}$ are considerably larger than their natural size $1/2m \simeq 0.5\,\mathrm{GeV}^{-1}$, see e.g. the review [7]. It therefore appears mandatory to perform a complete $\mathcal{O}(q^4)$ calculation to see whether such logarithms are present and to find a more accurate bound on the size of the remainder Δ_R.

2. The tool to systematically calculate *all* corrections to a given order is chiral perturbation theory (CHPT). It amounts to a systematic expansion around the chiral limit in terms of two small parameters related to the quark masses and the external momenta. To have a consistent power counting in the presence of baryons, the latter have to be treated as very heavy (static) sources, i.e. non–relativistically. We follow here the systematic SU(2) approach developed in Ref.[8]. In the framework of heavy baryon CHPT and to order q^4, we have to consider pion loop diagrams with at most one insertion from the dimension two pion–nucleon Lagrangian and local contact terms from $\mathcal{L}_{\pi N}^{(4)}$ accompanied by a priori unknown coefficients, the so–called low-energy constants (LECs). These we are estimating by resonance exchange since not enough precise data exist yet to pin them all down. However, previous calculations have already shown that this approach of treating the LECs is fairly accurate as long as no big cancellations appear (for details, see [7]).

Consider first the possible contact term contributions. Tree–level $\Delta(1232)$-exchange is independent of the off-shell parameter Z entering the $\pi N\Delta$-vertex. Using the empirically well satisfied large N_c coupling constant relation $g_{\pi N\Delta} = 3 g_{\pi N}/\sqrt{2}$ and the Goldberger–Treiman relation $g_A = g_{\pi N} F_\pi / m$, we find

$$\Delta_R^{(\Delta)} = \frac{g_A^2\, M_\pi^4}{4 m_\Delta (m_\Delta^2 - m^2)} \left(2 + \frac{m}{m_\Delta} \right) = 0.58\,\mathrm{MeV} \tag{6}$$

in good agreement with the estimate of Brown et al.[1]. Furthermore, $N^*(J^P = \frac{1}{2}^\pm)$ nucleon resonance exchange gives

$$\Delta_R^{(N^*)} = 0 \ , \tag{7}$$

verifying the general argument given in [1] within the chiral effective field theory approach. Here, by chiral symmetry requirements the pion coupling is of vector/axial-vector type

for the parity odd/even spin-$\frac{1}{2}$ resonances. The situation concerning the scalar–isoscalar meson exchange is somewhat more complex. Using the lowest order effective Lagrangians consistent with chiral symmetry [9]

$$\mathcal{L}_{NS} = g_S \, S \, \bar{\Psi}\Psi \ , \quad \mathcal{L}_{S\pi} = S \, [c_m \, \mathrm{Tr}\chi_+ + c_d \, \mathrm{Tr}(u_\mu u^\mu)] \tag{8}$$

in the conventional notation (S denotes the scalar and Ψ the nucleon field), we find

$$\Delta_R^{(S,2)} = 0 \ . \tag{9}$$

While the term $\sim c_m$ cancels in Δ_R to all orders if the scalar meson propagator is chirally expanded, the one $\sim c_d$ is proportional to $q \cdot q' = M_\pi^2 - t/2$ and thus vanishes at the Cheng–Dashen point. Only a four–derivative scalar meson–pion coupling

$$\mathcal{L}_{S\pi} = c_{4d} \, S \ \mathrm{Tr}(D \cdot u)^2 \tag{10}$$

would make a non–vanishing contribution to Δ_R. Since from the phenomenological side essentially nothing is known about the strength of such a vertex we can only give an estimate based on dimensional arguments. The low–energy constant c_3 (defined in Eq.(13) below) has been determined from low-energy πN data, $c_3 \simeq -4\,\mathrm{GeV}^{-1}$ [10] and its value can be understood from combined Δ and scalar–isoscalar exchange, the latter contributing at most $c_3^{(S)} = -1\,\mathrm{GeV}^{-1}$ [7]. Furthermore, we have $c_3^{(S)} = 2 g_S c_d / M_S^2$ and thus the four-derivative–scalar contribution to Δ_R takes the form

$$\Delta_R^{(S,4)} = 4 M_\pi^4 \, \frac{c_{4d} \, g_S}{M_S^2} = 2 M_\pi^4 \left| \frac{c_{4d}}{c_d} \, c_3^{(S)} \right| \ . \tag{11}$$

Notice that the sign of c_{4d} is not fixed, we have chosen it to give a positive contribution to Δ_R and thus we can obtain an upper bound on the remainder. Assuming now that each derivative D_μ is suppressed by $1/4\pi F_\pi$, i.e. the typical scale of chiral symmetry breaking, we get $|c_{4d}/c_d| = 0.73\,\mathrm{GeV}^{-2}$. This gives $\Delta_R^{(S,4)} = 0.55\,\mathrm{MeV}$. Allowing for a factor of two uncertainty, we arrive at

$$\Delta_R^{(S)} \simeq 1.1\,\mathrm{MeV} \ \ . \tag{12}$$

We now turn to the calculation of the one-loop graphs with exactly one insertion from $\mathcal{L}_{\pi N}^{(2)}$. The dimension two chiral πN Lagrangian has the form [7, 8]

$$\mathcal{L}_{\pi N}^{(2)} = \bar{N} \Big\{ c_1 \, \mathrm{Tr} \, \chi_+ + c_2 \, (v \cdot u)^2 + c_3 \, u_\mu u^\mu + c_4 \, [S^\mu, S^\nu] u_\mu u_\nu \Big\} N + 1/m - \text{terms} \tag{13}$$

where the terms not shown explicitly are the ones which by Lorentz invariance have fixed coefficients, like e.g. $\bar{N} \, D^2/2m \, N$. Let us make one general remark on the calculation. Whereas the quantities of interest here, $\sigma(2M_\pi^2)$ and $F_\pi^2 \, \bar{D}^+(0, 2M_\pi^2)$, derive from Lorentz invariant functions, calculations in the heavy baryon formalism require the choice of a specific kinematical frame. To evaluate the scalar form factor we choose the Breit frame with $v \cdot (p - p') = 0$. Furthermore, to the order we are working $\bar{D}^+(0, 2M_\pi^2)$ is given by the spin and isospin averaged πN scattering amplitude in the center–of–mass frame with

pion energy $v \cdot q = v \cdot q' = M_\pi^2/2m$ and momentum transfer $(q-q')^2 = 2M_\pi^2$, disregarding the nucleon pole diagrams. We have checked that corrections due to this necessary choice of frame are of order q^5 and higher in all cases. Omitting further calculational details, let us simply enumerate the results for the various contributions:

1) The terms of the form M_π^4/mF_π^2 and $g_A^4 M_\pi^4/mF_\pi^2$ contributing to $F_\pi^2\,\bar{D}^+(0,2M_\pi^2)$ all sum up to zero. For the latter this seems to contradict the result of ref.[5], cf. Eq.(5). However, only the non–analytic pieces (in the quark mass) in the scattering amplitude and the scalar form factor must agree with the relativistic calculation and this is obviously the case here. For the finite analytic loop pieces, these do not have to be equal in both calculations and they can be matched onto each other by appropriate counter terms (see ref.[8]). In fact, in all cases where in the relativistic calculation one has one–loop functions which have a cut starting at $t_0 = 4m^2$ in the dispersive representation, like for the nucleon isoscalar electromagnetic and isovector axial radii or the Goldberger–Treiman discrepancy, one finds a finite piece from the pertinent one–loop graphs. In the heavy baryon approach, these cuts are moved to infinity and thus the one–loop graphs have no finite piece.

2) The terms of the form $g_A^2 M_\pi^4/mF_\pi^2$ give exactly the same total sum of contributions to $F_\pi^2\,\bar{D}^+(0,2M_\pi^2)$ and to $\sigma(2M_\pi^2)$, namely $3g_A^2 M_\pi^4(\pi-4)/(128\pi^2 m F_\pi^2)$, thus
$$\Delta_R^{(g_A^2/m)} = 0 \ . \tag{14}$$
This agrees with the finding in ref.[5].

3) Consider now the loops with exactly one insertion proportional to $c_{1,2,3,4}$ (see Fig.1). First, one has to take care of the renormalization $F \to F_\pi$ in the order q^2 terms $\sim c_1$ (with F the pion decay constant in the chiral limit). Both the isoscalar πN amplitude at the Cheng–Dashen point and the scalar form factor of the nucleon at $t = 2M_\pi^2$ contain pieces of the type $M_\pi^4 \ln M_\pi$, but the resulting expressions are *identical* for both
$$\begin{aligned} &F_\pi^2\,\bar{D}^+(0,2M_\pi^2)^{(c_i-loop)} = \sigma(2M_\pi^2)^{(c_i-loop)} \\ &= \frac{M_\pi^4}{16\pi^2F_\pi^2}\Big\{3c_1\Big(8\ln\frac{M_\pi}{\lambda}+\pi-2\Big)+c_2\Big(-2\ln\frac{M_\pi}{\lambda}-\frac{\pi}{4}+\frac{7}{6}\Big)-6c_3\ln\frac{M_\pi}{\lambda}\Big\} \end{aligned} \tag{15}$$
which is quite an astonishing result[#2] and it implies that
$$\Delta_R^{(c_i-loop)} = 0 \quad . \tag{16}$$

[#2]Note that individually these contributions to the isoscalar πN amplitude and $\sigma(2M_\pi^2)$ in Eq.(15) give numerically about -11 MeV (for $\lambda \simeq 1$ GeV and $c_{1,2,3}$ taken from [10]). It is therefore not unreasonable to expect the remainder Δ_R coming from the c_i-loop graphs to be of similar magnitude.

4) We have not explicitly calculated all strangeness effects in Δ_R but can estimate it from the $K\eta$ loop contribution to $\sigma(2M_\pi^2)$ at order M_π^4 [6]

$$\sigma(2M_\pi^2)^{(K\eta-loop)} \simeq \frac{5M_\pi^4}{384\pi F_\pi^2 M_K} \simeq 0.4\,\mathrm{MeV} \simeq -0.04\cdot\sigma(2M_\pi^2)^{(c_i-loop)}\ . \tag{17}$$

We thus conjecture that the $K\eta$ loop contribution to Δ_R is bounded by some fraction of 1 MeV.

3. To summarize, we have calculated the remainder at the Cheng–Dashen point, $\Delta_R = M_\pi^4\,C_R$, to order q^4 in heavy baryon chiral perturbation theory. We have proven that C_R has *no* chiral logarithm and thus it is finite in the chiral limit. To this order, only local contact terms contribute to the remainder. Our estimate based on the complete q^4 CHPT calculation with the low energy constants of the pertinent counter terms saturated via resonance exchange is

$$\Delta_R \approx 2\,\mathrm{MeV}\ , \tag{18}$$

which we consider a conservative upper bound. As conjectured in ref.[3] the remainder Δ_R indeed does not play any role in the extraction of the σ–term from the πN data considering the present status of accuracy of these data in the threshold region.

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Figure

Fig.1 Loop diagrams which lead to the result given in Eq.(15). The circle–cross denotes an insertion from $\mathcal{L}_{\pi N}^{(2)}$ proportional to $c_{1,2,3}$. Full, broken and wavy lines represent nucleons, pions and the external scalar source, respectively. Subsets of diagrams which add up to zero are not shown.

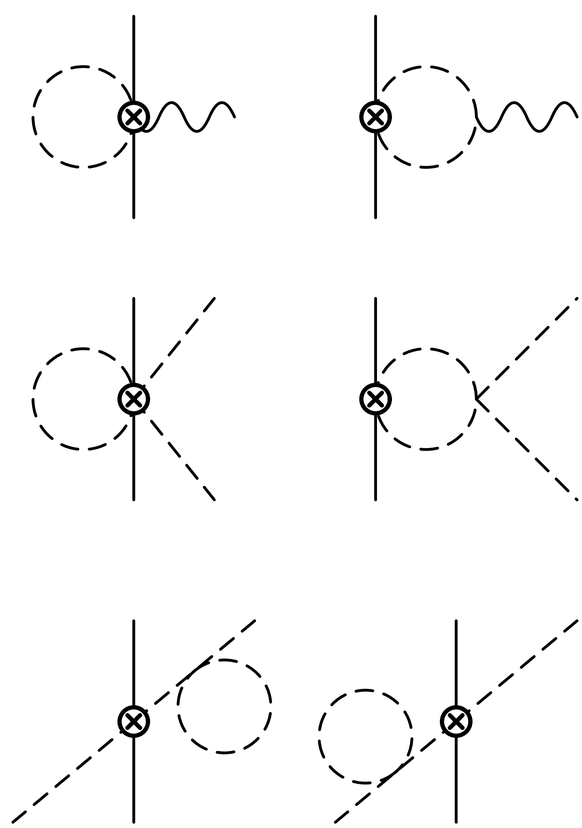

Figure 1